# Derivation of Kirchhoff diffraction integrals for soft and hard surfaces of perfectly electric conductor half-planes 

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## ABSTRACT


#### Abstract

In this study, by applying the Kirchhoff diffraction integral to the problem of the soft and hard-surface half-planes, two different diffraction integrals are obtained. To obtain the diffracted fields by the soft and hard-surface half planes the edge point method is used. For the uniform field intensities, Signum function and Fresnel function are used. In the comparison of the obtained diffracted fields, the results are interpreted by means of the plots as well. Similarities and differences between the obtained diffracted fields are discussed for some parameters. It is concluded that in some regions diffracted wave by a soft surface has an intensity higher than the diffracted field intensity corresponding to the hard surface. In some regions, it is vice versa. At the critical points that are related to the incident angle, they have the same value.


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## 1. INTRODUCTION

Wave diffraction by a half-plane has been an important fundamental investigation field for decades. Sommerfeld obtained first the solution to this problem in 1896 [1]. An alternate way to solve the problem is the multipole expansion method [2]. By employing parabolic cylinder functions, Lamb obtained the exact solution to the scattering by a perfectly electric conductor (PEC) half-sheet [3, 4]. The same approach was also used by Friedlander [5] and Chambers [6]. The geometrical theory of diffraction (GTD) was introduced by Keller in 1962 [7]. The GTD is based on the asymptotic evaluation of the diffracted fields originating from an edge at high-frequencies. Jull worked on the plane wave diffraction by a PEC half plane in an anisotropic plasma [8]. The electromagnetic diffraction phenomenon in an anisotropic medium is studied by Williams [9]. Przezdziecki worked on the diffraction by a half-plane that is perpendicular to the distinguished axis of the uniaxially anisotropic medium [10]. Fisanov studied the diffraction of cylindrical waves originating from a line source by a half-plane in an anisotropic plasma [11]. Kirchhoff employed the integral solution of the Helmholtz equation for the waves scattered by opaque aperture and obtained Kirchhoff diffraction integral which is used for scattering and electromagnetic propagation problems [12].

Antenna characteristics, frequency bands, and scattering characteristics of soft and hard surfaces can be investigated using the Kirchhoff diffraction integral. Umul introduced the modified

[^0]diffraction theory of Kirchhoff. In the study, Kirchhoff's diffraction theory is reinterpreted, and a new form of diffraction integral proposed using the modified theory of physical optics (MTPO) is used to solve the diffraction problem of a semi-infinite contour [13]. Umul studied a new representation of the Kirchhoff diffraction integral in which the integral boundaries are rearranged [14]. Kara investigated the scattering phenomenon of inhomogeneous plane waves by two half-planes having different properties [15]. Umul examined the diffraction of evanescent plane waves by a resistive half-plane [16]. Kara investigated the scattering of a plane wave incident on a cylindrical parabolic reflector with an arbitrary complex angle [17]. Kara worked on the diffraction of a plane wave by an aperture between two isorefractive media [18]. Stamnes used a hybrid technique by combining the asymptotic methods with the numerical integration technique [19]. Kara studied the fields of a line source diffracted by a cylindrical parabolic Perfectly Electric Conducting (PEC) reflector by employing the scattering integral of the modified theory of physical optics (MTPO) [20]. Zernov and Darmon studied the refinement of the Kirchhoff approximation to the scattered elastic fields [21]. Macdonald examined the effect caused by an obstacle on electric waves [22]. Ufimtsev studied on a new insight into the classical Macdonald physical optics approximation [23]. Khizhnyak et al. investigated the structure of edge-dislocation waves originating in plane wave diffraction by a half plane [24]. Kara examined asymptotic evaluation of
scattering of inhomogeneous plane waves by a PEC half plane [25]. Kyoung et al. studied on the far field detection of terahertz near field enhancement of sub-wavelength slits using Kirchhoff integral formalism [26].

In this study, by applying the Kirchhoff diffraction integral to the problem of soft and hard half-planes, two different diffraction integrals will be obtained. The novelty of this study is the investigation of the diffracting behaviours of soft and hard surfaces by using the Kirchhoff diffraction integral and determining the difference between them considering the halfplane diffraction. In the material and method section, theoretical derivations are obtained by employing the Kirchhoff diffraction integral. In the numerical results section, diffracted fields obtained for the soft and hard surfaces are discussed by interpreting their plots.

## 2. MATERIAL AND METHOD

A perfectly electric conductor half plane is considered. It is located on $y=0$ plane, $x \in(0, \infty)$ and $z \in(-\infty, \infty)$. The geometry is given in Figure 1 where $\varphi_{0}$ is the angle of incidence, $\alpha$ is the scattering angle and $P$ is the observation point located at $(x, y, z)$. Kurchhoff's diffraction integral is written as in Equation (1).
$u(P)=-\frac{1}{4 \pi} \iint[u \nabla G-G \nabla u] \cdot \vec{n} d S^{\prime}$
where $G$ is the Green function which is expressed as $G=\exp (-j k R) /$ $R$, and u is the total field. $\vec{n}$ is the normal vector to the surface. $R$ is the distance between the scattering point $x^{\prime}$ and the observation point. $k$ is the wave number. A soft (smooth) surface reflects the incident wave in only one direction. However a hard (rough) surface scatters the incoming wave in more than one direction, but the magnitude of the waves in the reflection direction is greater than that of the waves in other directions. It is known that the softness or hardness of a surface depends on the frequency and the angle of incidence. For a hard surface, the derivative of the total field $u$ on the surface is zero, while For the soft surface, $u$ is zero. Letting the incident plane wave $u_{i}$ given in Figure 1.


Fig. 1. The geometry of the diffraction.
$u_{i}=\widehat{e_{z}} u_{0} e^{j k(x \cos \varphi+y \sin \varphi)}$
where $u_{0}$ will be taken as unity for simplicity. Kirchhoff's integral in Equation (1) becomes,
$u(P)=\frac{1}{4 \pi} \iint G \nabla u \cdot \vec{n} d S^{\prime}$
where normal vector $\vec{n}$ is in $y$-direction and, Kirchhoff diffraction integrai is obtained by following the Equations (4) through (8).
$\nabla u \cdot \vec{n}=\left(\frac{\partial u}{\partial x} \overrightarrow{e_{x}}+\frac{\partial u}{\partial y} \overrightarrow{e_{y}}+\frac{\partial u}{\partial z} \overrightarrow{e_{z}}\right) \cdot \overrightarrow{e_{y}}=\frac{\partial u}{\partial y}$
$\nabla u \cdot \vec{n}=\frac{\partial u}{\partial n}=\frac{\partial u}{\partial y}$
$u(P)=\frac{1}{4 \pi} \iint G \frac{\partial u}{\partial y} d S^{\prime}$
$u(P)=\frac{j k}{2 \pi} \int_{x^{\prime}=0}^{\infty} \int_{z^{\prime}=-\infty}^{\infty} \sin \varphi_{0} e^{j k x \prime \cos \varphi_{0}} \frac{e^{-j k R}}{R} d x^{\prime} d z^{\prime}$
$u(P)=\frac{j k \sin \varphi_{0}}{2 \pi} \int_{x^{\prime}=0}^{\infty} \int_{z^{\prime}=-\infty}^{\infty} \frac{e^{j k\left(x^{\prime} \cos \varphi_{0}-R\right)}}{R} d x^{\prime} d z^{\prime}$
where the phase varies according to $\exp \left(j k x^{\prime} \cos \varphi_{0}\right)$, and $\exp (-j k R) / R$ means that the wave is spherical. We are dealing with the first integral term in Equation (8) which has the form of,
$\int_{a}^{\infty} f(x) e^{-j k g(x)} d x$
Which can be rewritten as;
$\int_{a}^{\infty} f(x) \frac{g^{\prime}(x)}{g^{\prime}(x)} e^{-j k g(x)} d x$
By letting,
$u=\frac{f(x)}{g \prime(x)}$
$d v=g^{\prime}(x) e^{-j k g(x)}$
$d u=\frac{f^{\prime}(x) g^{\prime}(x)-f(x) g^{\prime \prime}(x)}{\left[g^{\prime}(x)\right]^{2}}$
$v=-\frac{1}{j k} e^{-j k g(x)}$
Applying the integration by parts to Equation (10), we write,
$\int_{a}^{\infty} f(x) e^{-j k g(x)} d x=-\frac{1}{j k} \frac{f(a)}{g^{\prime}(a)} e^{-j k g(a)}-$
$\frac{j}{j k} \int_{a}^{\infty} e^{-j k g(x)} \frac{f^{\prime}(x) g^{\prime}(x)-f(x) g \prime \prime(x)}{\left[g^{\prime}(x)\right]^{2}} d x$
Integral on the right side of Equation (15) can be ignored due to the $k^{2}$ term in the denominator that makes the term tend to zero. Therefore, the diffraction integral is obtained in Equation (16).
$E_{d}=\int_{a}^{\infty} f(x) e^{-j k g(x)} d x \cong-\frac{1}{j k} \frac{f(a)}{g^{\prime}(a)} e^{-j k g(a)}$
Equation (16) is the result of the Edge Point Method. Applying this result to Equation (8), we write the amplitude function;
$f(x)=\frac{1}{R}$
and the phase function
$g\left(x^{\prime}\right)=x^{\prime} \cos \varphi_{0}-R$
The derivative of the phase function is written as;
$\frac{d g}{d x^{\prime}}=g^{\prime}\left(x^{\prime}\right)=\cos \varphi_{0}-\frac{\partial R}{\partial x^{\prime}}$
$g^{\prime}\left(x^{\prime}\right)=\cos \varphi_{0}-\frac{|x-x \prime|}{R}$
where;
$\frac{|x-x| \mid}{R}=\cos \alpha$
Substituting Equation (21) into Equation (20) we get,
$g^{\prime}\left(x^{\prime}\right)=\cos \varphi_{0}-\cos \alpha$
The diffraction integral becomes,
$u_{d}(P)=\frac{1}{j k} \frac{j k \sin \varphi_{0}}{2 \pi} \int_{-\infty}^{\infty} \frac{\sin \varphi_{0}}{\cos \varphi_{0}-\cos \alpha_{e}} \frac{e^{-j k R e}}{R_{e}} d z^{\prime}$
where $\cos \alpha$ at the edge point is
$\cos \alpha_{e}=-\frac{x}{R_{e}}$
and $R_{e}$ is the value of $R$ at the edge point as shown in Figure 2.


Fig. 2. The geometry of edge diffraction.
As a result, we obtain the diffraction integral for a soft surface as
$u_{d}(P)=-\frac{1}{2 \pi} \int_{-\infty}^{\infty} \frac{\sin \varphi_{0}}{\cos \varphi_{0}-\cos \alpha_{e}} \frac{e^{-j k R_{e}}}{R_{e}} d z^{\prime}$
Finally, the integral in Equation (25) can be approximated to;
$u_{d}(P)=-\frac{1}{2 \pi} \frac{\sin \varphi_{0}}{\cos \varphi_{0}-\cos \alpha_{e}} H_{0}^{2}(k \rho)$
where $H_{0}^{2}(k \rho)$ is the zeroth order of the second kind Hankel function which is expressed as;
$H_{0}^{2}\left(k R_{e}\right) \cong \sqrt{\frac{2}{\pi}} e^{\frac{j \pi}{4}} \frac{e^{j k \rho}}{\sqrt{k \rho}}$
where, $R_{e}=\rho$ from Figure 2 .
Now, we will consider the Kirchhoff diffraction integral for a hard surface where the derivative of the total field on the surface is zero. Taking $\nabla u=0$ in Equation (1) written;
$u(P)=-\frac{1}{4 \pi} \iint u \nabla G \cdot \vec{n} d S^{\prime}$
By writing;
$\frac{\partial G}{\partial n}=\frac{\partial G}{\partial R} \frac{\partial R}{\partial n}$
Optained below Equation;
$\frac{\partial G}{\partial n}=-\left(j k+\frac{1}{R}\right) \frac{e^{-j k R}}{R} \frac{\partial R}{\partial n}$
Ignoring $\frac{1}{R}$ in Equation (30);
$\frac{\partial G}{\partial n} \cong-j k \frac{e^{-j k R}}{R} \frac{\partial R}{\partial n}$
Equation (28) is rewritten as below;
$u(P)=\frac{j k}{4 \pi} \iint u \frac{e^{-j k R}}{R} \frac{\partial R}{\partial n} d S^{\prime}$
where;
$R=\sqrt{\left(x-x^{\prime}\right)^{2}+y^{2}+\left(z-z^{\prime}\right)^{2}}$
$\frac{\partial R}{\partial n}=\frac{y}{R}=\sin \alpha$
where $n=y$;
$u(P)=2 \frac{j k}{4 \pi} \int_{x^{\prime}=0}^{\infty} \int_{z^{\prime}=-\infty}^{\infty} e^{j k x \prime \cos \varphi_{0}} \sin \alpha \frac{e^{-j k R}}{R} d z^{\prime} d x^{\prime}$
As a result, the diffraction integral is concluded as;
$E_{d}=u(P)=-\frac{1}{2 \pi} \int_{z^{\prime}=-\infty}^{\infty} \frac{\sin \alpha_{e}}{\cos \varphi_{0}-\cos \alpha_{e}} \frac{e^{-j k R_{e}}}{R_{e}} d z^{\prime}$
Finally;
$E_{d}=u(P)=-\frac{1}{2 \pi} \frac{\sin \alpha_{e}}{\cos \varphi_{0}-\cos \alpha_{e}} H_{0}^{2}(k \rho)$
By comparing Equation (26) and Equation (37), we see that only $\sin \varphi_{0}$ in Equation (26) is replaced by $\sin \alpha_{e}$ in Equation (37). The uniform expression of the diffraction expressions obtained in Equation (26) and Equation (37) for hard and soft half planes are written as;
$E_{d-\text { hard }}=$
$-2 e^{-\frac{j \pi}{4}} \frac{\sin \alpha_{e}}{\sin \left(\varphi_{0}\right)}\left[\cos \left(\frac{\varphi_{0}+\varphi_{e}}{2}\right) \operatorname{sign}\left(t_{1}\right) F\left[\left|t_{1}\right|\right] e^{j k \rho \cos \left(\varphi_{0}+\varphi_{e}\right)}+\right.$
$\left.\cos \left(\frac{\varphi_{0}-\varphi_{e}}{2}\right) \operatorname{sign}\left(t_{2}\right) F\left[\left|t_{2}\right|\right] e^{j k \rho \cos \left(\varphi_{0}-\varphi_{e}\right)}\right]$
$E_{d-s o f t}=$
$-2 e^{-\frac{j \pi}{4}}\left[\cos \left(\frac{\varphi_{0}+\varphi_{e}}{2}\right) \operatorname{sign}\left(t_{1}\right) F\left[\left|t_{1}\right|\right] e^{j k \rho \cos \left(\varphi_{0}+\varphi_{e}\right)}+\right.$
$\left.\cos \left(\frac{\varphi_{0}-\varphi_{e}}{2}\right) \operatorname{sign}\left(t_{2}\right) F\left[\left|t_{2}\right|\right] e^{j k \rho \cos \left(\varphi_{0}-\varphi_{e}\right)}\right]$
(39)

Respectively, where;
$t_{1}=-\sqrt{2 k \rho \cos \left(\frac{\varphi_{0}+\varphi_{e}}{2}\right)}$
$t_{2}=-\sqrt{2 k \rho \cos \left(\frac{\varphi_{0}-\varphi_{e}}{2}\right)}$
$\operatorname{sign}(x)$ is the signum function which is equal to 1 for $x>0$, and -1 otherwise. $F[x]$ is the Fresnel functiondefined by,
$F[x]=\frac{e^{\frac{j \pi}{4}}}{\sqrt{\pi}} \int_{x}^{\infty} e^{-j t^{2}} d t$

## 3. NUMERICAL RESULTS

In this section, examined the behaviour of the diffracted fields caused by soft and hard-surface half-planes. The observation distance $\rho$ is taken as $6 \lambda$ where $\lambda$ is the wavelength. If it is selected greater than $6 \lambda$ the amplitude decreases, but increases if the observation distance is taken less than $6 \lambda$. The observation angle varies in $[0-2 \pi]$.

In Figure 3, diffracted fields for the soft and hard surfaces are plotted simultaneously. For the given values, and the angle of the incident plane wave is $\pi / 3$, it is observed that diffracted field intensity for the soft surface is greater than the intensity of the hard surface in the intervals of $\varphi \in[-\pi / 3, \pi / 3]$ and $\varphi \in$ $[2 \pi / 3,4 \pi / 3]$. For $\varphi \in\left[\frac{\pi}{3}, 2 \pi / 3\right]$ and $\varphi \in[4 \pi / 3,5 \pi / 3]$ intensity of the hard surface is greater than the intensity of the soft surface. These results show that between the region of the incident and reflected waves, i.e. $\varphi \in\left[\frac{\pi}{3}, \frac{2 \pi}{3}\right]$ whose general form is $\varphi \in$ $\left[\varphi_{0}, \pi-\varphi_{0}\right]$, the amplitude of the diffracted waves corresponding to the hard surface is greater than that of the soft surface. This is true for $\varphi \in\left[\pi+\varphi_{0}, 2 \pi-\varphi_{0}\right]$. In these regions relected field is dominant. However, outside these regions, where the diffracted field is dominant, the intensity of the waves corresponding to the soft surface is greater than that of the hard surface. At the critical points, that are $\varphi_{0}, \pi-\varphi_{0}, \pi+$ $\varphi_{0}$, and $2 \pi-\varphi_{0}$ they have the same value.

In Figure 4, diffracted fields for the soft and hard surfaces are plotted simultaneously. For the given values, and the angle of the incident plane wave is $\pi / 6$, diffracted field intensity for the hard surface is greater than that of the hard one in the intervals of $\varphi \in$ $\left[\varphi_{0}, \pi-\varphi_{0}\right]=\left[\frac{\pi}{6}, \pi-\frac{\pi}{6}\right] \quad$ and $\varphi \in\left[\pi+\varphi_{0}, 2 \pi-\varphi_{0}\right]=\left[\frac{7 \pi}{6}\right.$, $\left.\frac{11 \pi}{6}\right]$. However, in the other regions, the intensity of the soft surface is much greater than that of the hard surface due to the reasons given in Figure 3.


Fig. 3. Diffracted field intensities for soft and half-hard-surface planes.

Some observation values are used to observed difffracted field intensity variations fort he hard surface, and the results are given in Figure 5. It is observed that as the distance increases, the field intensity tends to decrease.


Fig. 4. Diffracted field intensities for soft and hard-surface planes.


Fig. 5. Diffracted field intensity variations according to observation distance for the hard-surface half-plane.

## 4. CONCLUSION

In this study, A half-plane is considered placing at $S=$ $\{(x, y, z) ; x \in(0, \infty), y=0, z \in(-\infty, \infty)\}$. Kirchhoff diffraction integral is used for both soft and hard-surface halfplanes. In the case of a soft-surface half-plane, the total field on the surface is zero. By considering this fact, only the Green function times the gradient of the incoming wave remains in the Kirchhoff diffraction integral. This integral is reduced to the one obtained in Equation (25) by employing the edge point method. Finally, the diffracted field for the soft surface is obtained in Equation (26). For the hard-surface half-plane situation, the derivative of the total field on the surface is zero. In that case, the incident field times the gradient of the Green function remains in the Kirchhoff diffraction integral. Solving the surface integral, we obtained the diffraction integral given in Equation (36). Obtained the diffracted field for the hard surface in Equation (37). Diffracted fields obtained in Equation (26) and Equation (37) are not uniform. Their uniform versions are given in Equations (37) and (38). Observed that the uniform diffracted field expressions in Equations (37) and (38) differ from each other in that the diffracted field for the soft surface is multiplied by $\sin \left(\alpha_{e}\right) /$ $\sin \left(\varphi_{0}\right)$ to obtain the diffracted field for the hard-surface halfplane.

Appendix
The MATLAB software code, used for the plots of Equations (38) and (39), can be given as below; $\mathrm{l}=0.001$;
k=2. *pi./l;
fi0 $=$ pi./6;
fi=0:0.01:2. ${ }^{*} \mathrm{pi}$;
rho=6.*1;
$\mathrm{y}=$ rho. ${ }^{*} \sin (\mathrm{fi})$;
alfae $=\operatorname{asin}(\mathrm{y} . / \mathrm{rho})$;
$\mathrm{t} 1=-\mathrm{sqrt}(2 . * \mathrm{k} . * \mathrm{rho} . * \cos ((\mathrm{fi} 0+\mathrm{alfae}) . / 2))$;
$\mathrm{t} 2=-\mathrm{sqrt}(2 . * \mathrm{k} . *$ rho. $* \cos ((\mathrm{fi} 0-\mathrm{alfae}) . / 2))$;
Ed_soft=-2.*exp(-
j.*pi./4).*[cos((fi0+alfae)./2).*sign(t1).*fres(abs(t1)).*exp(j.*k.* rho. ${ }^{\cos (f i+f i 0))+\cos ((f i 0-}$
alfae)./2).*sign(t2).*fres(abs(t2)).*exp(j.*k.*rho.*cos(fi-fi0))];
Ed_hard=-2.*exp(-
j.*pi./4).*[sin(alfae)./sin(fi0)].*[cos((fi0+alfae)./2).*sign(t1).*fre $\mathrm{s}(\operatorname{abs}(\mathrm{t} 1)) . * \exp \left(\mathrm{j} . * \mathrm{k} . *\right.$ rho. $\left.{ }^{*} \cos (\mathrm{fi}+\mathrm{fi} 0)\right)+\cos ((\mathrm{fi} 0-$
alfae)./2).*sign(t2).*fres(abs(t2)).*exp(j.*k.*rho.*cos(fi-fi0))];
polar(fi,abs(Ed_hard),'r');
hold on;
polar(fi,abs(Ed_soft),'b');
hold on;
clear;
The Fresnel function "fres (u)" used in MATLAB code is given as below;
function $\mathrm{y}=$ fres $(\mathrm{u})$;
$\mathrm{N}=1000$;
sum=0;
asinir=0;
usinir=u;
delta=(usinir-asinir)./N;
for $\mathrm{i}=0: \mathrm{N}$;
$\mathrm{t}=$ asinir+(i.*delta);
$\mathrm{g}=\exp \left(-\mathrm{j} . *\left(\mathrm{t} .^{\wedge} 2\right)\right.$ );
sum=sum +g ;
end
$\mathrm{y}=0.5-\left(\exp (\mathrm{j} . * \mathrm{pi} . / 4) . *\right.$ sum. ${ }^{\text {delta./sqrt(pi) }) ; ~}$

| Nomenclature |  |
| :--- | :--- |
| PEC | Perfectly electric conductor |
| GTD | Geometrical theory of diffraction |
| MTPO | Modified theory of physical optics |
| $G$ | Green function |
| P | Observation point |
| $x^{\prime}$ | Scattering point |
| $R$ | Distance between scattering and observation |
|  | points (m) |
| $R_{e}$ | $R$ value for the edge point |
| $u$ | Total field (V/m) |
| $u i$ | Incident field (V/m) |
| $\vec{n}$ | Normal vector to the surface |
| $k$ | Wave number |
| $\operatorname{sign}(x)$ | Signum function |
| $F[x]$ | Fresnel function |
| Greek Symbols |  |
| $\lambda$ | Wavelength (m) |
| $\alpha$ | Scattering angle $\left({ }^{\circ}\right)$ |
| $\alpha_{e}$ | Scattering angle at the edge point $\left({ }^{\circ}\right)$ |
| $\varphi_{0}$ | Angle of incidence $\left({ }^{\circ}\right)$ |

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